Méthodes approchées pour l'optimisation combinatoire multiobjectif

Xavier GANDIBLEUX[1] and Matthias EHRGOTT[2]

(1) LAMIH - Recherche Opérationnelle et Informatique
Université de Valenciennes et du Hainaut-Cambrésis
Le Mont Houy, F-59313 Valenciennes cedex 9 – FRANCE
Xavier.Gandibleux@univ-valenciennes.fr

(2) Department of Engineering Science
University of Auckland
Private Bag 92019, Auckland – NEW ZEALAND
m.ehrgott@auckland.ac.nz

Journées Franciliennes de Recherche Opérationnelle

30 novembre 2001 — Paris, France

Some practical multiobjective combinatorial pb.

- **Portfolio optimization** (2-4 objectives)
  → 'Bicriteria quadratic knapsack problems' (Steuer 1985)
- **Telecommunications** (2 or more objectives)
  → 'Multicriteria shorter path problem' (Thiongane et al. 2001)
- **Trip organization** (2-5 objectives)
  → 'Preference-based Multicriteria TSP ALS : TPP' (Godart 2001)
- **Vehicule routing problem** (2-7 objectives)
  → 'Vehicule routing problem with time windows' (El-Sherbeny 2001)
- **Airline crew scheduling** (2 objectives)
  → 'Bicriteria set partitioning problems' (Ehrgott and Ryan 2001)
- **Railway network infrastructure capacity** (2 objectives)
  → 'Bicriteria set packing problems' (Delorme et al. 2001)

Telecommunications

- ◦ Min cost
- ◦ Max quality (delay, length)

→ 'Multicriteria shorter path problem'

to solve : Exact methods (Martins 1984, Corley and Moon 1985)

Trip organization

- Min transport cost
- Min activity cost
- Min lodging cost
- Max attractivity activities
- Max attractivity lodging

→ 'Preference-based Multicriteria TSP ALS (Trip Planning Problem)'
to solve : Approximative Solution Methods (SA & TS)

Railway network infrastructure capacity

- Max number of trains
- Max robustness

→ 'Bicriteria set packing problems'

to solve : Approximative Solution Methods (GRASP)

Content

Introduction - Problem definition

- **Finite set** $A = \{a_1, \ldots, a_n\}$
- $X \subseteq 2^A$

Example

- A = edges of graph
- X = paths

Objective functions

- $S \in X$, $w_q : A \to \mathbb{Z}$ $q = 1, \ldots, Q$ **weight functions**
- $z^q(S) = \sum_{a \in S} w_q(a)$
- $z^q(S) = \max_{a \in S} w_q(a)$

Multiobjective combinatorial optimization problem

$$\text{“}\min_{S \in X}\text{”} \left(\mathbf{z^1(S)}, \ldots, \mathbf{z^Q(S)}\right) \qquad \text{(MOCO)}$$

Definition of optimal solution

- **Pareto optimality/efficiency**
  - $S \in X$ efficient if there is **no** $S' \in X$ such that $z^j(S') \leq z^j(S)$ $j = 1, \ldots, Q$ and $z^q(S') < z^q(S)$ for some q
  - $z(S) = (z^1(S), \ldots, z^Q(S))$ is called nondominated
  - Pareto optimal (efficient) solutions: E
- . . .

Representation of $S \in X$ as binary vector $x \in \{0,1\}^n$

$$x_i = \begin{cases} 1 & e_i \in S \\ 0 & \text{else} \end{cases}$$

(MOCO) is a discrete optimization problem, with

- n **variables** x_i, $i = 1, \ldots, n$,
- Q **objectives** z^j, $j = 1, \ldots, Q$
- m **constraints** of specific structure defining X

Supported and Nonsupported Efficient Solutions

Linear programming

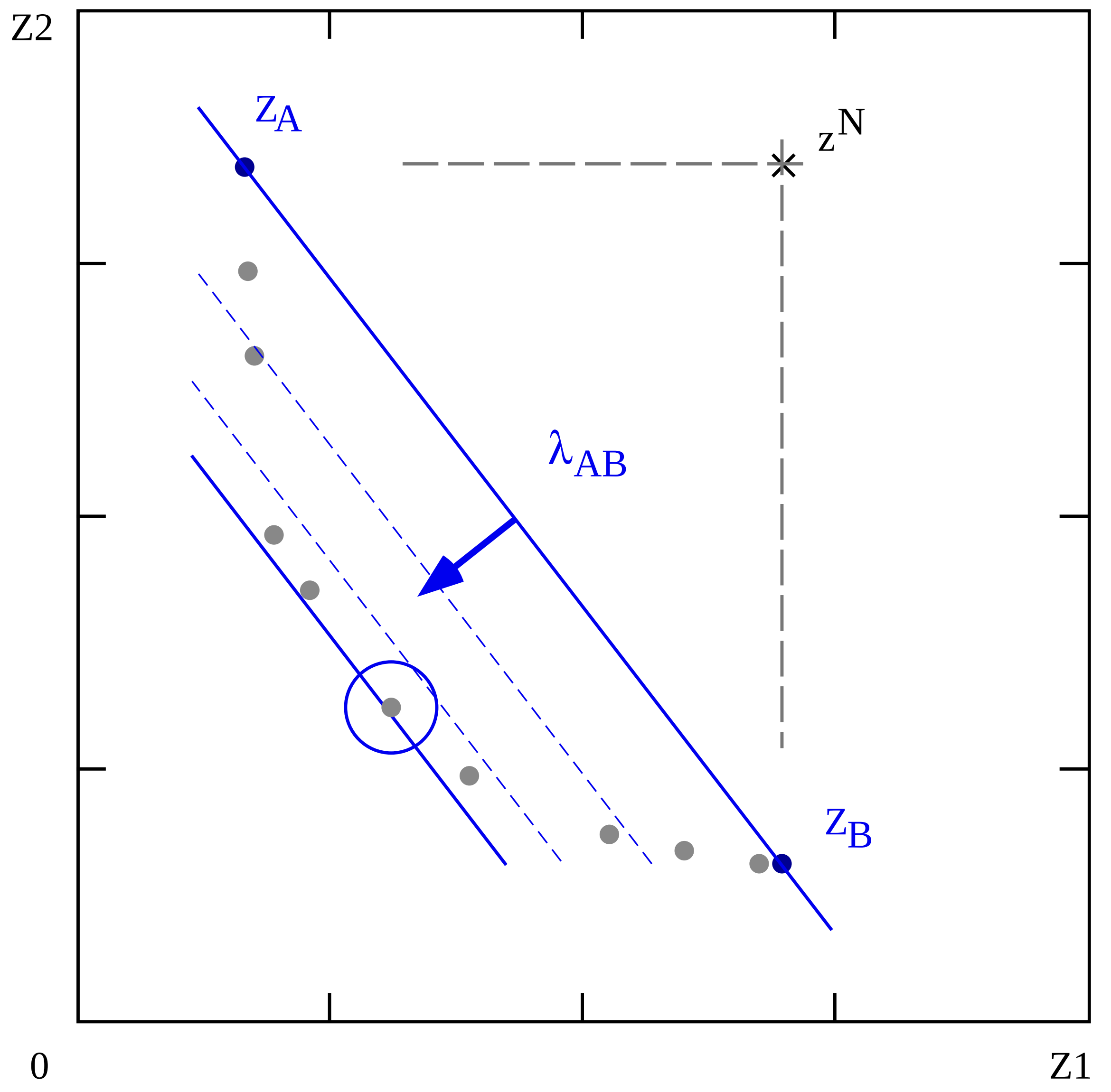
$$\min\{Cx : Ax = b, x \geq 0\}$$

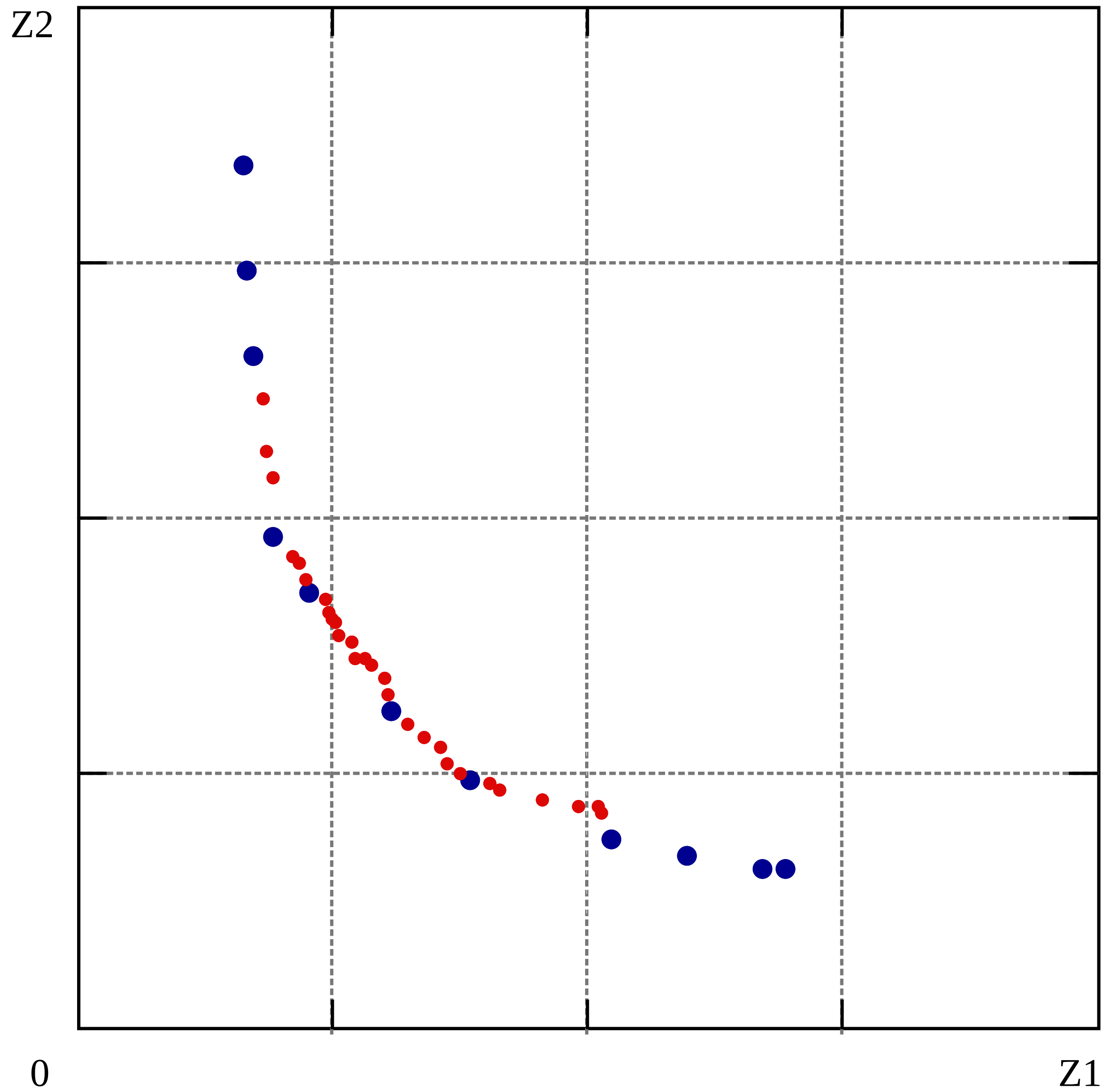
E is set of solutions of

$$\min\left\{\sum_{\mathbf{j=1,\dots,Q}} \lambda_{\mathbf{j}} \mathbf{c^j x} : \mathbf{Ax} = \mathbf{b}, \mathbf{x} \geq \mathbf{0}\right\}$$

with $0 < \lambda < 1 \quad \sum_{J=1}^{Q} \lambda_j = 1$

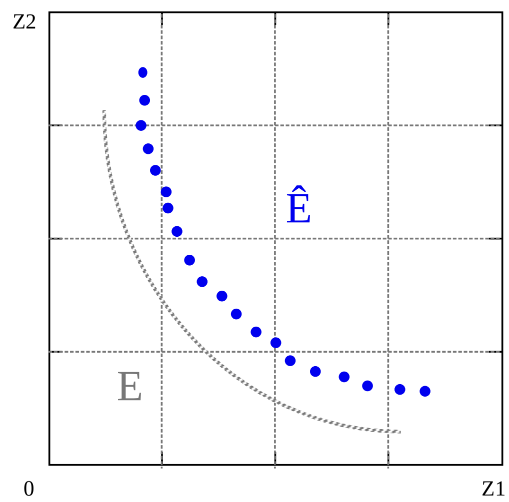
(MOCO) $\rightarrow$ supported efficient solutions SE, **nonsupported efficient solutions** NE exist


Z2
zA
zN
λAB
zB
0
Z1

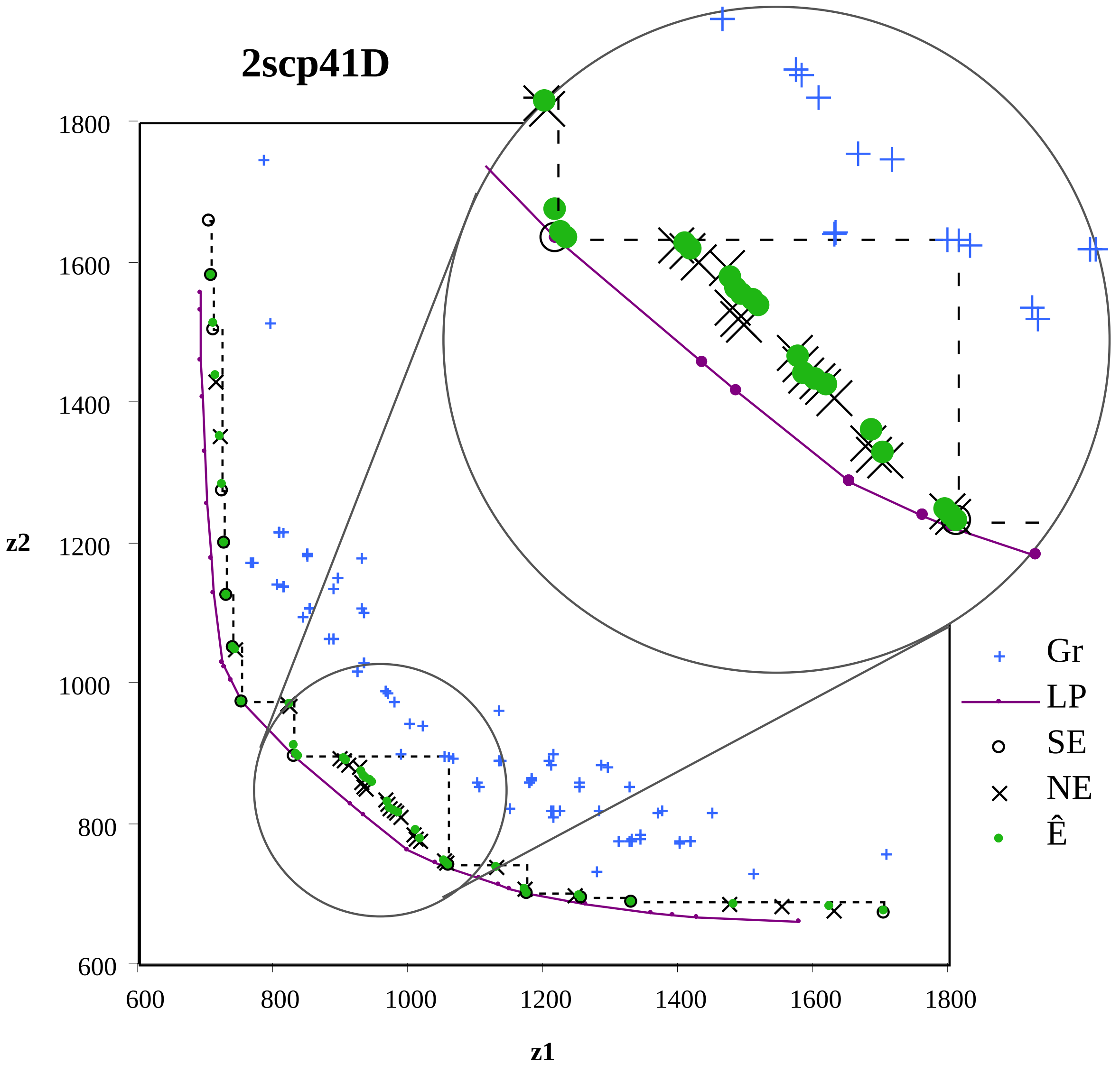

Z2
0
Z1

Approximation in a multiobjective context

- Good tradeoff between the
  - Quality of $\widehat{E} = \widehat{SE} \cup \widehat{NE}$
  - Time & memory requirements



- Measure of quality ?
  - coverage / uniformity / cardinality (Sayin 2000)
  - Bounds and bound sets (Ehrgott and Gandibleux 2001)
  - Discussion (Hansen and Jaszkiewicz 1998)


2scp41D
z2
1800
1600
1400
1200
1000
800
600
600
800
1000
1200
1400
1600
1800
z1
Gr
LP
SE
NE
Ê

Multiobjective Metaheuristics (MOMH)

- **genetic algorithms** (GA, Schaffer 1984)
- **neural networks** (NN, Malakooti 1990)
- **simulated annealing** (SA, Serafini 1992)
- **tabu search** (TS, Gandibleux 1996)

- **Evolutionary Algorithms**
- **Neighborhood Search Algorithms**

(1/2) : Evolutionary algorithms vs Neighborhood search algorithms

History	EA	1984 first algorithm: VEGA
	NSA	1992 Serafini's discussion of SA
Iteration mechanism	EA	Evolution operators (mutation, crossover)
	NSA	Explicit use of neighborhood notion
Generality	EA	Universal algorithms, ready to use
	NSA	Rather a method than an algorithm
Scientific communities	EA	Computer scientists and engineers
	NSA	Operational researchers
Problems investigated	EA	Bi-objective, continuous variables, non-linear functions, no constraints
	NSA	Bi-objective with discrete variables, linear functions, linear constraints (MOCO)

(2/2) : Evolutionary algorithms vs Neighborhood search algorithms

Real applications	EA	Considerable applications on real situations
	NSA	Very few real applications
Attractivity	EA	An important number of publication
	NSA	Few works in comparison with EA
Comparative studies	EA	Several comparative studies
	NSA	Quasi absence of comparative studies
Places of discussion	EA	EMO conf., specialized GA-EA conf.
	NSA	MCDM and MOPGP conf.

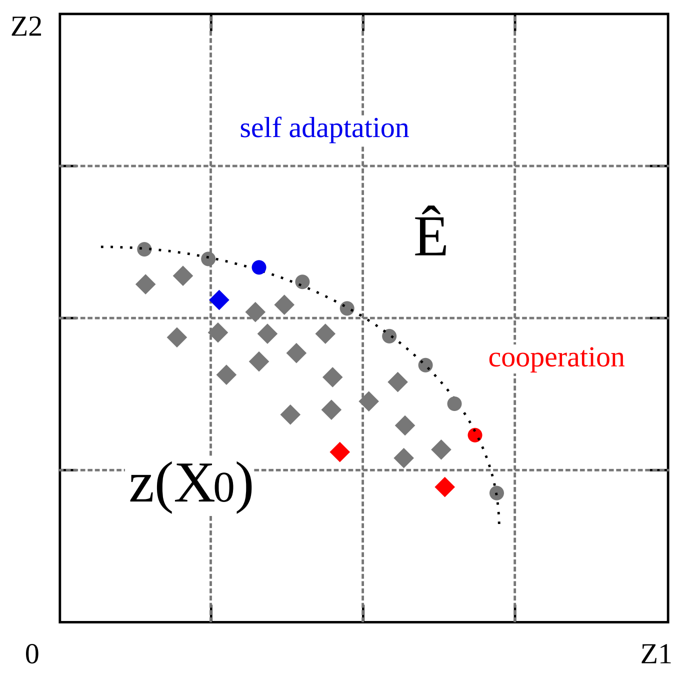
MultiObjective MetaHeuristics :

Evolutionary Algorithms Wave

Evolutionary Algorithms Wave

- Evolutionary Algorithms
- Vector Evaluated Genetic Algorithm by Schaffer (1984)
- The Multiobjective Evolutionary Algorithms Wave
- Major Issues for MOEA
- Significant MOEA
- MOEA and MOCO Problems

Evolutionary Algorithms

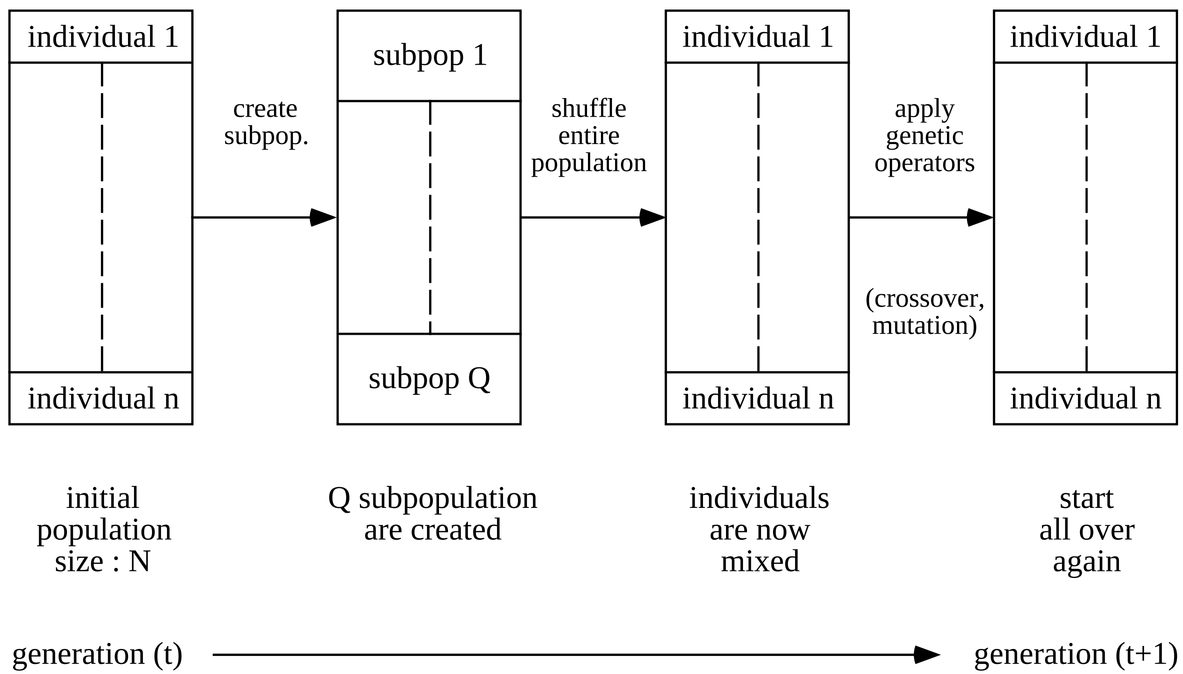


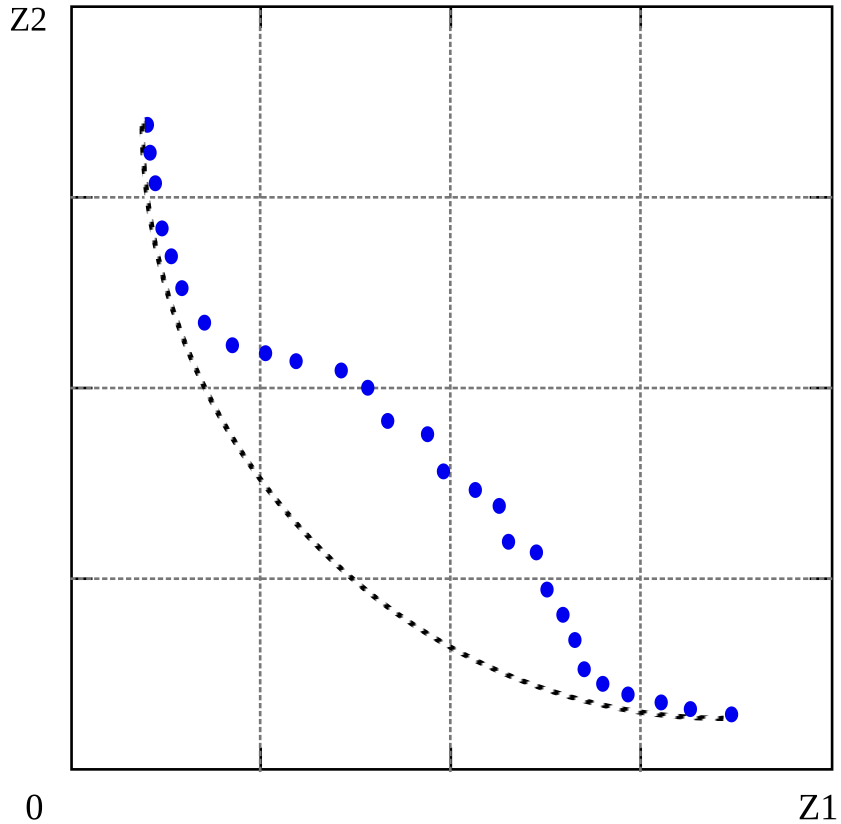
1. **Initial population** X_0
2. **Self adaptation**, i.e. independent evolution
3. **Cooperation**, i.e. exchange of information between individuals

⇒ **Parallel process where the whole population contributes to the evolution process to generate** $\widehat{E}$

Vector Evaluated Genetic Algorithm by Schaffer (1984)

- Extension of GENESIS to Vector Evaluated GA (VEGA)
- Non Pareto based method
- Generation process (parallel selection)


individual 1
individual n
create
subpop.
subpop 1
subpop Q
shuffle
entire
population
individual 1
individual n
apply
genetic
operators
(crossover,
mutation)
individual 1
individual n
initial
population
size : N
Q subpopulation
are created
individuals
are now
mixed
start
all over
again
generation (t)
generation (t+1)

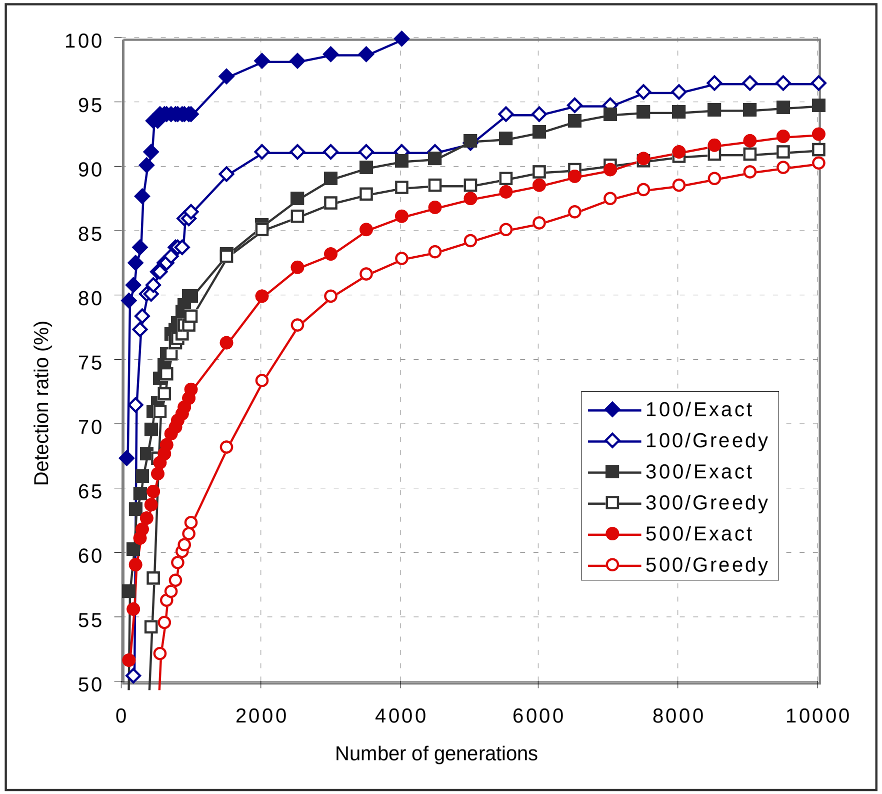

Z2
0
Z1

MOEA : Two central questions

1. **Uniform convergence**
   How to accomplish both fitness assignment and selection, in order to guide the search toward the efficient frontier?
2. **Uniform distribution**
   How to maintain a diversified population in order to avoid premature convergence and find a uniform distribution of solutions along the efficient frontier?

ranking / niching / sharing

MOEA : Elite solutions



- Exact SE solutions
- Some greedy solutions

(Gandibleux et al 2001)

Significant MOEA

- **Multiple Objective Genetic Algorithm** (MOGA93)
  by Fonseca and Fleming, 1993.
- **Nondominated Sorting Genetic Algorithm** (NSGA)
  by Srinivas and Deb, 1994.
- **Niched Pareto Genetic Algorithm** (NPGA)
  by Horn, Nafpliotis and Goldberg, 1994.
- **Multiple Objective Genetic Algorithm** (MOGA95)
  by Murata and Ishibuchi, 1995.
- **Strength Pareto Evolutionary Algorithm** (SPEA)
  by Zitzler and Thiele, 1998.
- **Pareto Archived Evolution Strategy** (PAES)
  by Knowles and Corne, 1999.

Several surveys

C. M. Fonseca and P. J. Fleming.
An Overview of Evolutionary Algorithms in Multiobjective Optimization. *Evolutionary Computation*, 3(1):1–16, Spring 1995.

C.A. Coello.
A comprehensive survey of evolutionary-based multiobjective optimization techniques. *Knowledge and Information Systems*, accepted, 1999.

C.A. Coello.
An updated survey of GA-based multiobjective optimization techniques. *ACM Computing Surveys*, 32(2):109–143, 2000.

C.A. Coello.
EMO repository. `http://www.lania.mx/~ccoello/EMOO/`

D. Jones, S.K. Mirrazavi, and M. Tamiz.
Multi-objective meta-heuristics: An overview of the current state-of-the-art. Technical report, University of Portsmouth, UK, 2000.

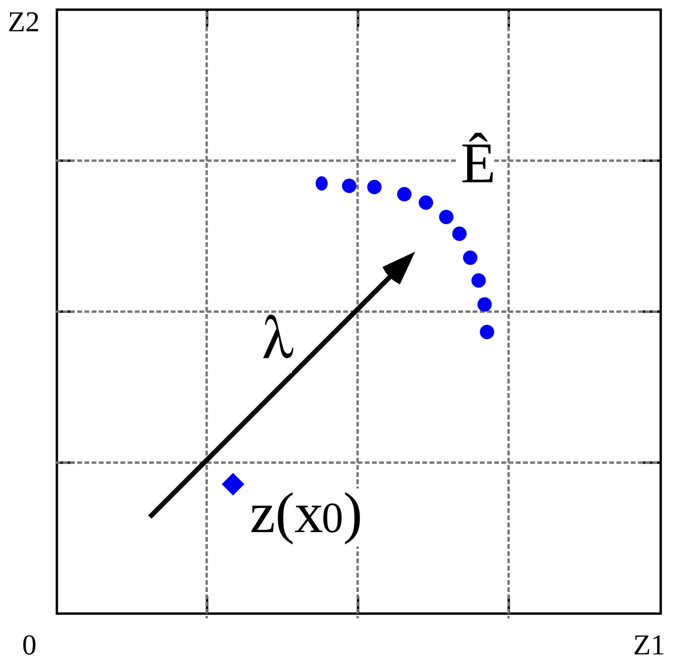
MultiObjective MetaHeuristics :

The simulated annealing wave

The simulated annealing wave

- The simulated annealing
- Multiobjective Simulated Annealing by Ulungu (1992)
- Pareto Simulated Annealing by Czyzak (1996)
- Multiobjective Simulated Annealing by Engrand (1997), revised by Parks (1999)
- Others Simulated Annealing based methods

MOSA92 by Ulungu, 1992



- **Initial solution** x_0
- **Neighbourhood structure** $\mathcal{N}(x_0)$
- **Search directions** λ
- **Local aggregation** mechanism $S(z(x), \lambda)$

$\Rightarrow$ Sequential process in the objective space Z

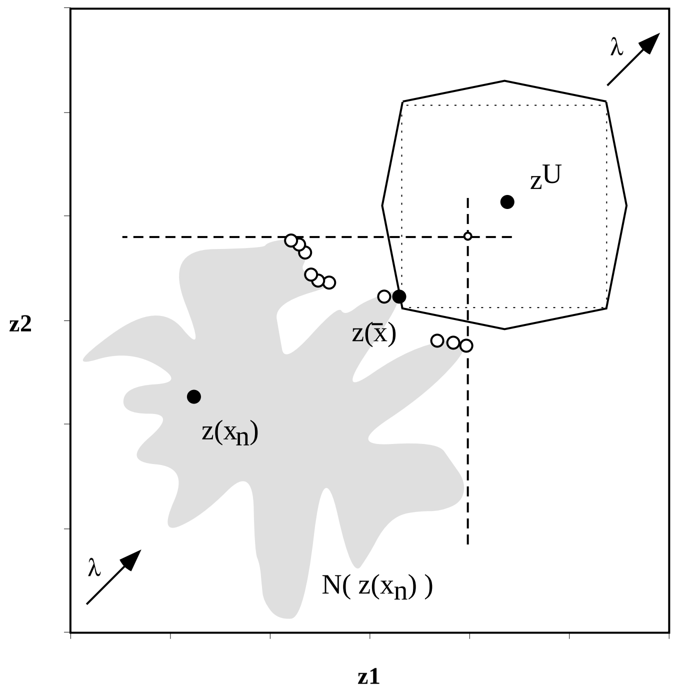
MultiObjective MetaHeuristics :

The tabu search wave

The tabu search wave

- The tabu search
- MultiObjective Tabu Search by Gandibleux (1996)
- MultiObjective Tabu Search by Hansen (1997)
- Tabu Search and Weighted Tchebycheff metric by Sun (1997)
- MultiObjective Tabu Search by Baykasoglu (1999)
- Others Tabu Search based methods in brief

MOTS96 by Gandibleux, 1996



- **Initial solution** x_0
- **Neigh. structure** $\mathcal{N}(z(x_0))$
- **Search directions** λ
- **Tabu process**
- **Reference point**
- **Local aggregation** mechanism $s(z(x), z^U, \lambda)$
- **Tabu memory to browse** Z

MultiObjective MetaHeuristics :

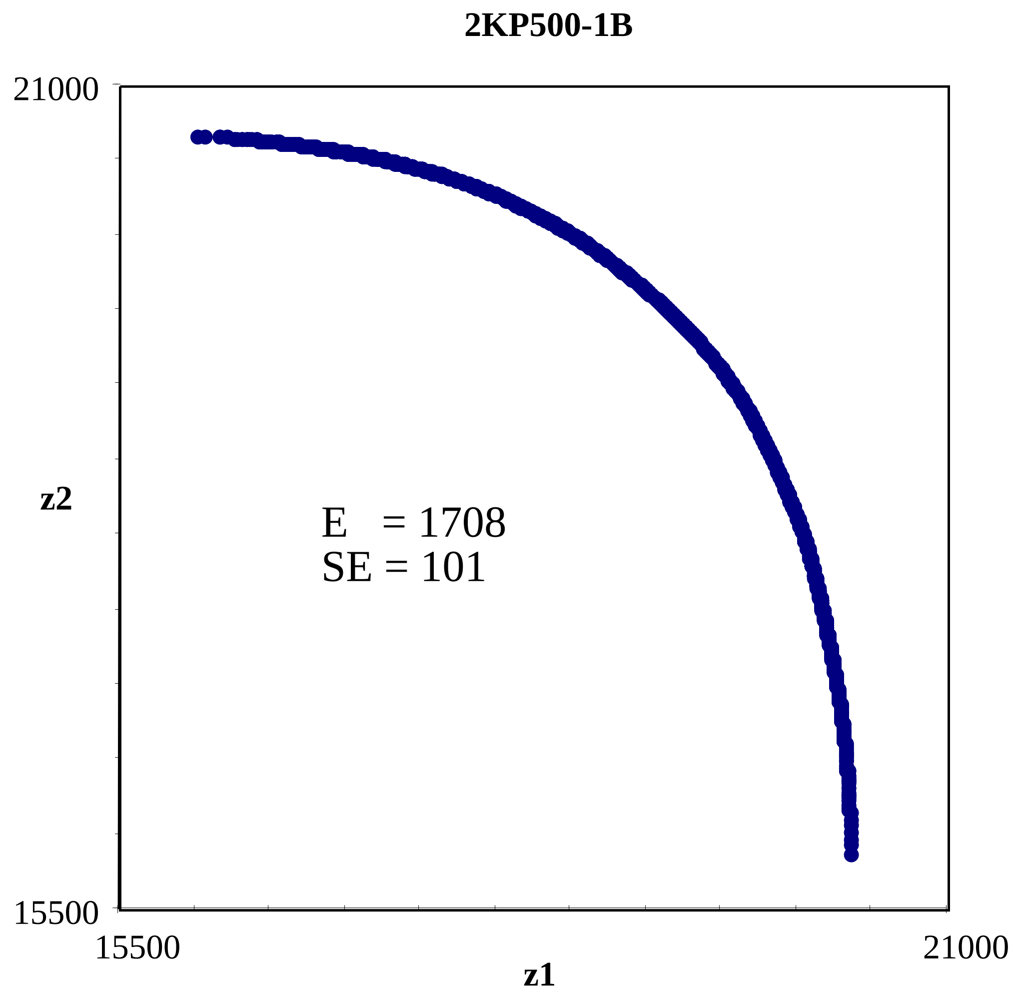
Other waves

Other waves

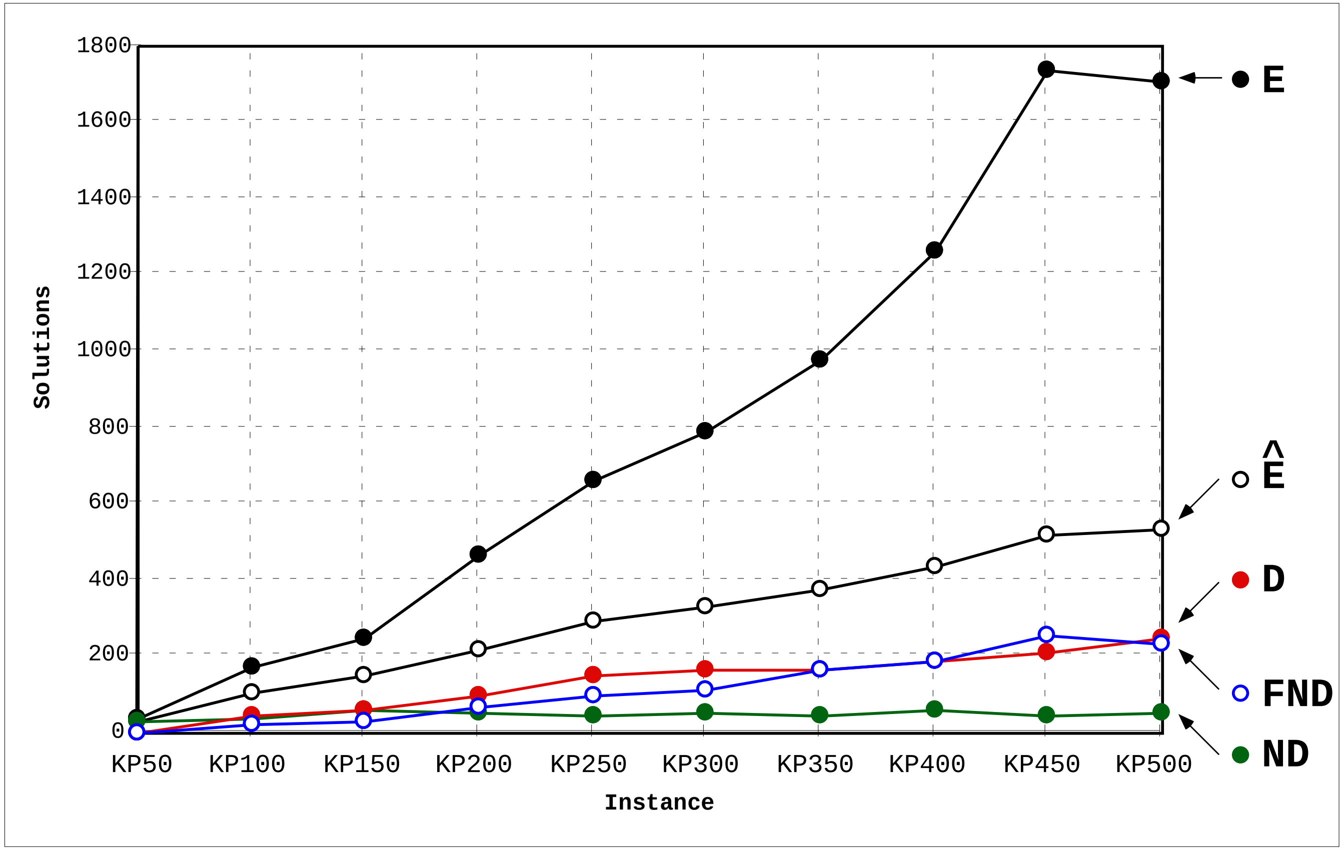
- Neural network (2)
- GRASP (2)
- Ants system (3)
- Scatter search (1)

Efficient solutions and decision-aid

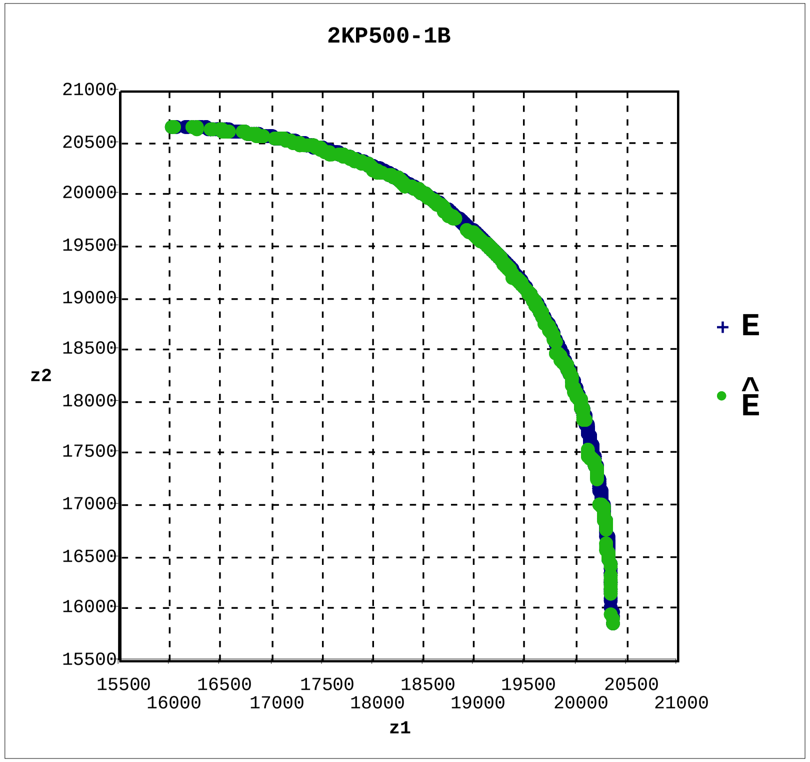
All efficient solutions (! or ?)

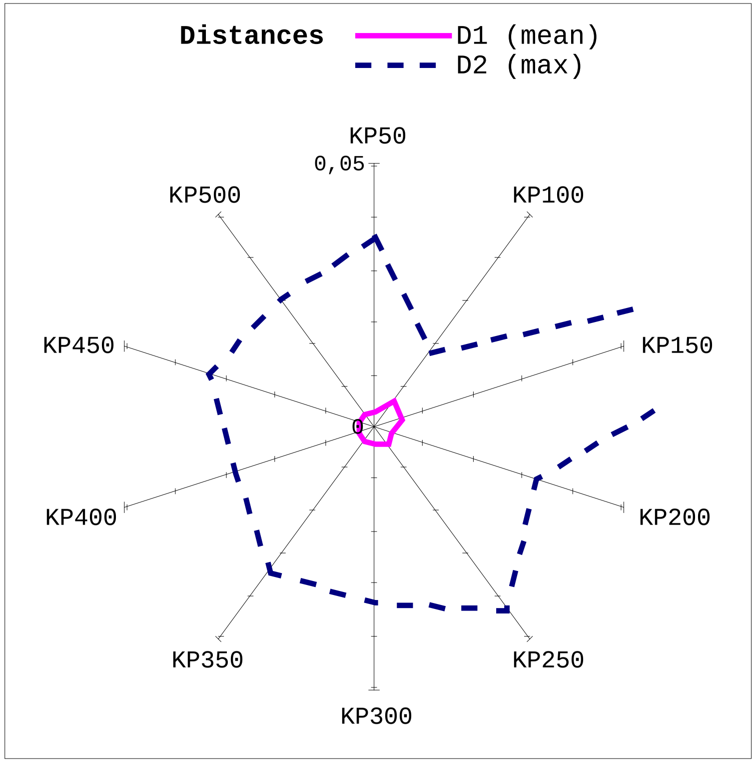


A quick approximation

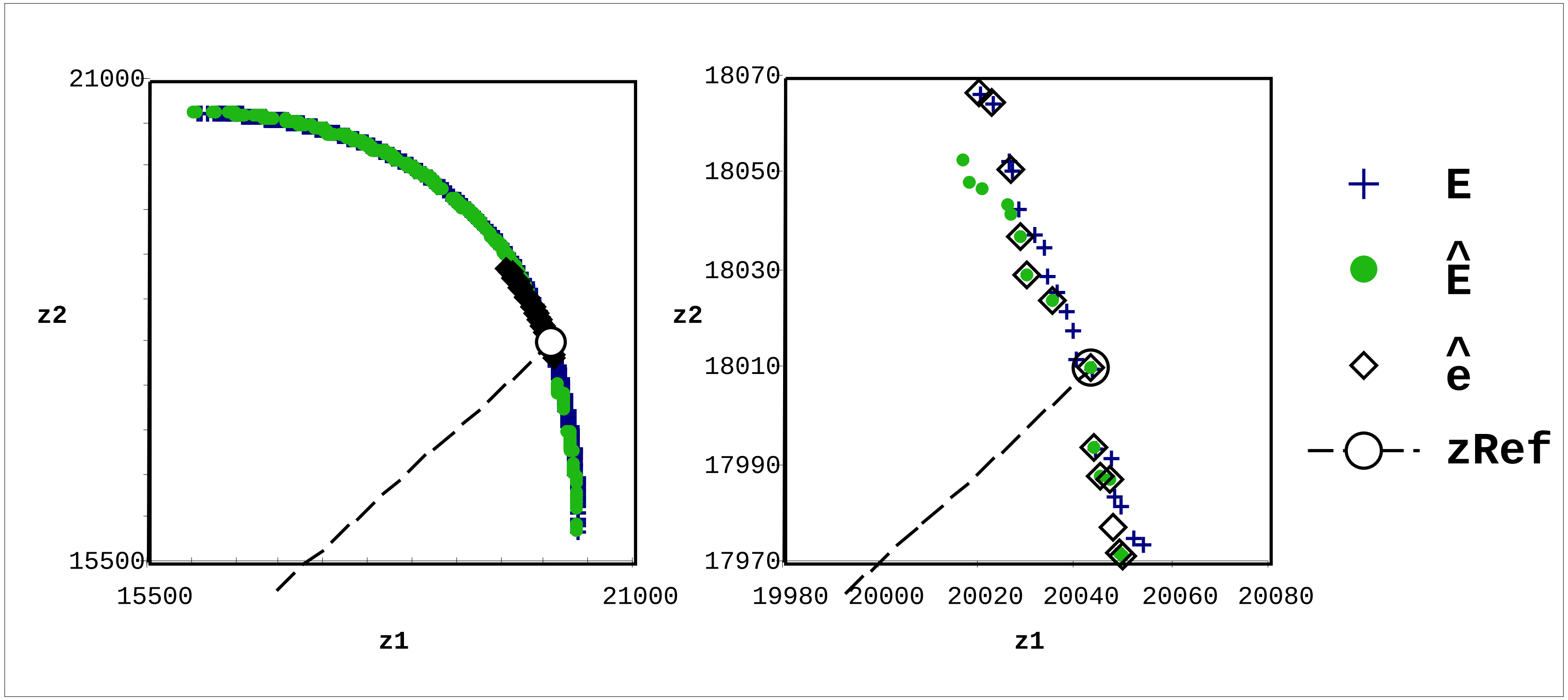


Qualitative and quantitative analysis





An interactive mode



Some informations

Ressources : Papers

- **Approximative resolution methods for multiobjective combinatorial optimization** (Working paper in preparation).
- M. Ehrgott, X. Gandibleux (2000) **A Survey and annotated Bibliography of Multiobjective Combinatorial Optimization**; *OR Spektrum*, volume 22, 2000, pages 425-460.
- M. Ehrgott, X. Gandibleux (Eds) (2002) **Multiple criteria optimization : state of the art annotated bibliographic surveys**; *Kluwer Academic Publishers*. 500 pages. To appear (summer 2002).

www.univ-valenciennes.fr/ROAD/XavierG/xgPapers.html

Ressources : MCDM Numerical Instances Library

- MultiObjective Assignment Problem
- MultiObjective Knapsack Problem
- MultiObjective Set Covering Problem
- MultiObjective Traveling Salesman Problem
- Test Problems for Multiobjective Optimizers
- … (we are waiting for your instances)

www.univ-valenciennes.fr/ROAD/MCDM/

Ressources : PM2O

- Groupe de travail ROADEF
  'Programmation mathématique multi-objectif (PM20)'.
- ROADEF'2002, Special session 'PM2O'.
- Next PM2O meeting, May 2002, Angers.

www.li.univ-tours.fr/pm2o/